

Does HM mechanism work in string theory?

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Abstract: The correspondence principle offered a unique opportunity to test the Horowitz and Maldacena mechanism at correspondence point "the centre of mass energies around $(M_s/(g_s)^2)$ ". First by using Horowitz and Maldacena proposal, the black hole final state for closed strings is studied and the entropy of these states is calculated. Then, to consider the closed string states, a copy of the original Hilbert space is constructed with a set of operators of creation/annihilation that have the same commutation properties as the original ones. The total Hilbert space is the tensor product of the two spaces $H_{right} \otimes H_{left}$, where in this case $H_{left/right}$ denotes the physical quantum states space of the closed string. It is shown that closed string states can be represented by a maximally entangled two-mode squeezed state of the left and right spaces of closed string. Also, the entropy for these string states is calculated. It is found that black hole entropy matches the closed string entropy at transition point. This means that our result is consistent with correspondence principle and thus HM mechanism in string theory works. Consequently the unitarity of the black hole in string theory can be reconciled. However Gottesman and Preskill point out that, in this scenario, departures from unitarity can arise due to interactions between the collapsing body and the infalling Hawking radiation inside the event horizon and information can be lost. By extending the Gottesman and Preskill method to string theory, the amount of information transformation from the matter to the state of outgoing Hagedorn radiation for closed strings is obtained. It is observed that information is lost for closed strings.

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I. INTRODUCTION

Recently Horowitz and Maldacena have suggested a mechanism to reconcile the unitarity of black hole evaporation [1]. In this proposal, there were three different Hilbert spaces that belong to degrees of freedom of matter, incoming, and outgoing radiation. The total state of a black hole is a direct product of matter state and the entangled states of the states inside and outside of the event horizon. When a black hole begins to evaporate, the matter state will be in a maximally entangled state with incoming Hawking radiation. To describe the unknown effects of quantum gravity, an additional unitary transformation S is introduced. The outside state of a black hole is easily obtained by projecting this entangled state on the total state of the black hole (HM mechanism)[1]. The question arises is that if there is a possibility for testing HM mechanism in string theory. Using the correspondence principle we can examine this idea at transition point.

The correspondence principle in string theory has provided us with a convincing picture of the evolution of a black hole at the last stages of its evaporation [2]. As the black hole shrinks, it eventually reaches the correspondence point at the centre of mass energies around $(M_s/(g_s)^2)$, and makes a transition to a configuration dominated by a highly excited long string (string ball). Here g_s and M_s are the string coupling and string mass respectively. At this point, the production cross section, entropy and other properties of string balls will match the production cross section, entropy and other properties of black holes[2–5]. We can obtain the final state of black hole by using HM proposal and derive the entropy for this black hole. Then we will compare this entropy with string ball entropy at correspondence point. If these entropies are the same, HM mechanism works and the unitarity of the black hole can be reconciled. However Gottesman and Preskill point out that, in this scenario, departures from unitarity can arise due to interactions between the collapsing body and the infalling Hawking radiation inside the event horizon. The amount of information loss when a black hole evaporates depends on the extent to which these interactions are entangling[5].

The important question is that if string model is sufficient to solve the information-loss problem. Using the Gottesman and Preskill method[5], we calculate the information transformation from the matter to the state of outgoing Hawking radiation for closed strings. If information transformation is complete, we can solve the information loss paradox in string theory.

The outline of the paper is as the following. In section II we obtain the entangled states on outside and inside Hilbert spaces of black hole for closed strings .Then we study the entangled states on inside and matter Hilbert spaces of the black hole for strings in section III . Next we obtain the final state of black hole for closed strings in section IV. We compare the entropy carried out by closed strings with string ball entropy at transition point in section V. Finally we calculate the information transformation from collapsing matter to outside of black hole in section VI .We summarize our results in section VII.

II. THE BLACK HOLE STATES FOR CLOSED STRING

In this section we discuss the evaporation of Schwarzschild black holes into closed strings and show that the ground state for these strings is a maximally entangled state on outside and inside Hilbert spaces of black holes .

First let us consider a bosonic closed string in flat space time and then extend it to this space time. A closed string moving in a d dimensional flat space-time is described by d two-dimensional fields $x^\mu(\sigma, \tau)$, $\mu = 0, 1, \dots, d-1$ with $0 \leq \sigma \leq 2\pi$ and can be expand in modes as[6]:

$$X^\mu = x_0^\mu + \sqrt{2\alpha'} p^\mu \tau + i\sqrt{\alpha'/2} \sum_{n \neq 0} \frac{e^{-in\tau}}{n} (\alpha_n^\mu e^{in\sigma} + \bar{\alpha}_n^\mu e^{-in\sigma}) \quad (1)$$

where $l_s = \sqrt{2\alpha'} = \frac{1}{\sqrt{\pi T}}$ is the fundamental string length and T is string tension. The center of mass coordinate is x_0^μ and string momentum is p^μ . The commutation relations satisfied by the various operators are:[7]

$$[\bar{\alpha}_m^\mu, \bar{\alpha}_n^\nu] = m\delta_{m+n,0}g^{\mu\nu}, [\alpha_m^\mu, \alpha_n^\nu] = m\delta_{m+n,0}g^{\mu\nu}, [\bar{\alpha}_m^\mu, \alpha_n^\nu] = 0 \quad (2)$$

where $g^{\mu\nu}$ is the metric in d dimensional space-time.

Now we consider excited closed string fields in black hole space-time .The stationary four dimensional Schwarzschild black hole with regarding internal space is represented by the metric [8]:

$$ds^2 = -(1 - \frac{2M_{BH}}{r})dt^2 + (1 - \frac{2M_{BH}}{r})^{-1}dr^2 + r^2d\Omega^2 + g_{ij}dx^i dx^j \quad (3)$$

where M_{BH} is the mass of black hole .At $r = 2M_{BH}$,the Schwarzschild black hole has an event horizon . Here, we assume a direct product structure with an internal space parameterized by x^i , which has the metric g_{ij} which can be taken to be flat.

In the black hole space-time ,the physical degrees of freedom dynamics for bosonic closed strings is governed by the equation:

$$(-g)^{1/2} \frac{\partial}{\partial x^\mu} [g^{\mu\nu} (-g)^{1/2} \frac{\partial}{\partial x^\nu}] X^i = 0, i = 1, \dots, d-1 \quad (4)$$

for the transversal coordinates of black hole observers, which is also valid for inertial coordinates. In Kruskal coordinates the metric of black hole becomes[8]:

$$\begin{aligned} ds^2 &= -2M_{BH} \frac{e^{-\frac{r}{2M_{BH}}}}{r} d\bar{u}d\bar{v} + r^2 d\Omega^2 + g_{ij} dx^i dx^j \\ \bar{u} &= -4M_{BH} e^{-u/4M_{BH}}, \bar{v} = -4M_{BH} e^{-v/4M_{BH}} \\ u &= t - r^*, v = t + r^*, r^* = -r - 2M_{BH} \ln |r - 2M_{BH}| \end{aligned} \quad (5)$$

Since the Killing vector in Kruskal coordinate is given by $\frac{\partial}{\partial \bar{u}}$ on past horizon " H^- ", the positive frequency normal mode solution in Kruskal coordinate is approximated by :

$$\bar{X}^i \propto e^{-iE_s \bar{u}} \quad (6)$$

where E_s is the string mass and $i=1,..d-1$ is the free index that is related to i th component of string. Using this fact that $\bar{v} = 0$ on past horizon " H^- " [9] we can estimate the original positive frequency normal mode on past horizon:

$$X^i \propto e^{-iE_s u} = \left(\frac{|\bar{u}|}{4M_{BH}} \right)^{-i4M_{BH}E_s} = \begin{cases} \left(-\frac{\bar{u}}{4M_{BH}} \right)^{-i4M_{BH}E_s} & (region I) \\ \left(\frac{\bar{u}}{4M_{BH}} \right)^{-i4M_{BH}E_s} & (region II) \end{cases} \quad (7)$$

where E_s is the energy of string. In Eq.(7), we can use the fact that $(-1)^{-i4M_{BH}E_s} = e^{4M_{BH}E_s}$. Using equation (7) we observe that the string states in the horizon satisfy the following condition[6]:

$$\begin{aligned} (X_{out}^i - \tanh r_{E_s} X_{in}^i) |BH\rangle_{in}^i \otimes_{out} &= 0 \\ \tanh r_{E_s} &= e^{-4M_{BH}E_s} \end{aligned} \quad (8)$$

which actually constitutes a boundary state. In fact, we can view Hawking radiation as the pair creation of a positive energy string that goes to infinity and a negative energy string that falls into the black hole. The pair is created in a particular entangled state. So the

Unruh state can be viewed as an entangled thermal state. Using the expansion in modes for closed strings (equation 1) we may write:

$$(\alpha_{n,out}^i - \tanh r_{E_s,\lambda} \alpha_{n,in}^{i\dagger}) |BH,n\rangle_{in}^i \otimes_{out} = 0 \text{ for } n \neq 0 \quad (9)$$

$$(\bar{\alpha}_{n,out}^i - \tanh r_{E_s,-\lambda} \bar{\alpha}_{n,in}^{i\dagger}) |BH,n\rangle_{in}^i \otimes_{out} = 0 \text{ for } n \neq 0 \quad (10)$$

$$\tanh r_{E_s,\lambda} = e^{-4M_{BH}E_s - 2in\lambda} \quad (11)$$

$$(x_{0,out}^i - x_{0,in}^i) |BH,0\rangle_{in}^i \otimes_{out} = 0 \text{ for } n = 0 \quad (12)$$

where we have defined the respective string coordinates up to a shift: $\sigma_{out} \equiv \sigma_{in} + \lambda[6, 10]$. Now, we assume that the Kruskal vacuum $|BH\rangle_{in}^i \otimes_{out}$ is related to the flat vacuum $|0\rangle_s$ by

$$|BH\rangle_{in}^i \otimes_{out} = F_1(\alpha_{n,out}^i, \alpha_{n,in}^{i\dagger}) F_2(\bar{\alpha}_{n,out}^i, \bar{\alpha}_{n,in}^{i\dagger}) |0\rangle_s \quad (13)$$

where F_1 and F_2 are some functions of closed string creation-annihilation operators to be determined later. These operators act on black hole vacuum and form string modes inside and outside of event horizon. It is clear that when the string does not intersect the horizon, the F transformations are trivial.

From $[\alpha_{n,out}^i, \alpha_{n,out}^{i\dagger}] = 1$, we obtain $[\alpha_{n,out}^i, (\alpha_{n,out}^{i\dagger})^m] = \frac{\partial}{\partial \alpha_{n,out}^{i\dagger}} (\alpha_{n,out}^{i\dagger})^m$ and $[\alpha_{n,out}^i, F] = \frac{\partial}{\partial \alpha_{n,out}^{i\dagger}} F$ [11]. Then using Eqs. (9) and (10), we get the following differential equations for F_1 and F_2 .

$$\left(\frac{\partial F_1}{\partial \alpha_{n,out}^{i\dagger}} - \tanh r_{E_s,\lambda} \alpha_{n,in}^{i\dagger} F \right) = 0 \quad (14)$$

$$\left(\frac{\partial F_2}{\partial \bar{\alpha}_{n,out}^{i\dagger}} - \tanh r_{E_s,-\lambda} \bar{\alpha}_{n,in}^{i\dagger} F \right) = 0 \quad (15)$$

The solution is given by

$$F_1 = e^{\tanh r_{E_s,\lambda} \alpha_{n,out}^{i\dagger} \alpha_{n,in}^{i\dagger}}, F_2 = e^{\tanh r_{E_s,-\lambda} \bar{\alpha}_{n,out}^{i\dagger} \bar{\alpha}_{n,in}^{i\dagger}} \quad (16)$$

By substituting Eq.(16) into Eq. (13) and by properly normalizing the state vector, we get

$$\begin{aligned} |BH,n\rangle_{in}^i \otimes_{out} &= N_{BH,n} e^{\tanh r_{E_s,\lambda} \alpha_{n,out}^{i\dagger} \alpha_{n,in}^{i\dagger}} e^{\tanh r_{E_s,-\lambda} \bar{\alpha}_{n,out}^{i\dagger} \bar{\alpha}_{n,in}^{i\dagger}} |0\rangle_s \\ &= \frac{1}{\cosh r_{E_s,\lambda}} \frac{1}{\cosh r_{E_s,-\lambda}} \sum_{m,\bar{m}} \tanh^m r_{E_s,\lambda} \tanh^{\bar{m}} r_{E_s,-\lambda} |n, m, \bar{m}\rangle_{in} \otimes |n, m, \bar{m}\rangle_{out} \end{aligned} \quad (17)$$

where $|n, m, \bar{m}\rangle_{in}$ and $|n, m, \bar{m}\rangle_{out}$ are orthonormal bases (normal mode solutions) for mode n that act on H_{in} and H_{out} respectively. $N_{BH,n}$ is the normalization constant. The total

ground state for black hole can be derived as:

$$|BH\rangle_{in}^i \otimes_{out} = \prod_n \frac{1}{\cosh^n r_{E_s, \lambda}} \frac{1}{\cosh^n r_{E_s, -\lambda}} \sum_{m, \bar{m}} \tanh^{nm} r_{E_s, \lambda} \tanh^{n\bar{m}} r_{E_s, -\lambda} \otimes |n, m, \bar{m}\rangle_{in} \otimes |n, m, \bar{m}\rangle_{out} \quad (18)$$

Eq. (18) states that the black hole state decomposes to many entangled states with different energies for closed string. On the other hand, this equation indicates that non-local physics would be required to transmit the string information outside the black hole, and that inside and outside the Hilbert spaces are dependent.

III. THE EVAPORATION OF BLACK HOLE TO CLOSED STRINGS

Previously it has been investigated that the field inside the event horizon can be decomposed into the collapsing matter field and that the advanced wave incoming from infinity has the same form as the Hawking radiation[9, 12]. We extend these calculations to string theory for closed strings. We introduce the advanced collapsing shell metric[9] in d dimensions at correspondence point as:

$$ds^2 = \begin{cases} -d\tau^2 + dr^2 + r^2 d\Omega^2 + g_{ij} dx^i dx^j & r < R(\tau) \\ -(1 - \frac{2M_{BH}}{r}) dt^2 + (1 - \frac{2M_{BH}}{r})^{-1} dr^2 + r^2 d\Omega^2 + g_{ij} dx^i dx^j & r > R(\tau) \end{cases} \quad (19)$$

The shell radius $R(\tau)$ is defined by[9]:

$$R(\tau) = \begin{cases} R_0 & \tau < 0 \\ R_0 - v\tau & \tau > 0 \end{cases} \quad (20)$$

where τ is the proper time. The advanced coordinates has been defined as[9, 12, 13]:

$$V = \tau + r - R_0, U = \tau - r + R_0 \quad (21)$$

Now we consider the closed string modes inside the black hole (on future horizon" H^+ "), which is given by[9]:

$$X^{+i} = \begin{cases} (-1)^{-i4M_{BH}M_s} (1 - \frac{\nu V}{(1-\nu)(R_0-2M_{BH})})^{-i4M_{BH}E_s} & V > \frac{(1-\nu)(R_0-2M_{BH})}{\nu} \text{matter wave function} \\ (1 - \frac{\nu V}{(1-\nu)(R_0-2M_{BH})})^{-i4M_{BH}E_s} & V < \frac{(1-\nu)(R_0-2M_{BH})}{\nu} \text{inside wave function} \end{cases} \quad (22)$$

Using Eq. (22) we observe that the string states in the horizon satisfy the following condition:

$$(X_{matter}^i - \tanh r_{E_s} X_{in}^i) |BH\rangle_{in}^i \otimes_{out} = 0$$

$$\tanh r_{E_s} = e^{-4M_{BH}E_s} \quad (23)$$

which actually constitutes a boundary state. Using the expansion in modes for closed strings (equation (23)) we write:

$$(\alpha_{n,matter}^i - \tanh r_{E_s,\lambda} \alpha_{n,in}^{i\dagger}) |BH,n\rangle_{in}^i \otimes_{out} = 0 \text{ for } n \neq 0 \quad (24)$$

$$(\bar{\alpha}_{n,matter}^i - \tanh r_{E_s,-\lambda} \bar{\alpha}_{n,in}^{i\dagger}) |BH,n\rangle_{in}^i \otimes_{out} = 0 \text{ for } n \neq 0 \quad (25)$$

$$\tanh r_{E_s,\lambda} = e^{-4M_{BH}E_s - 2in\lambda} \quad (26)$$

$$(x_{0,matter}^i - x_{0,in}^i) |BH,0\rangle_{in}^i \otimes_{out} = 0 \text{ for } n = 0 \quad (27)$$

The solution of equations (24) and (25) may be expressed as:

$$\begin{aligned} |BH,n\rangle_{matter}^i \otimes_{in} &= N_{BH,n} e^{\tanh r_{E_s,\lambda} \alpha_{n,matter}^{i\dagger} \alpha_{n,in}^{i\dagger}} e^{\tanh r_{E_s,-\lambda} \bar{\alpha}_{n,matter}^{i\dagger} \bar{\alpha}_{n,in}^{i\dagger}} |0\rangle_s \\ &= \frac{1}{\cosh r_{E_s,\lambda}} \frac{1}{\cosh r_{E_s,-\lambda}} \sum_{m,\bar{m}} \tanh^m r_{E_s,\lambda} \tanh^{\bar{m}} r_{E_s,-\lambda} |n,m,\bar{m}\rangle_{matter} \otimes |n,m,\bar{m}\rangle_{in} \end{aligned} \quad (28)$$

To obtain the total ground state inside the black hole ,we multiply the entangled states for different modes of closed string:

$$\begin{aligned} |BH\rangle_{matter}^i \otimes_{in} &= \prod_n \frac{1}{\cosh^n r_{E_s,\lambda}} \frac{1}{\cosh^n r_{E_s,-\lambda}} \sum_{m,\bar{m}} \tanh^{nm} r_{E_s,\lambda} \tanh^{n\bar{m}} r_{E_s,-\lambda} \otimes \\ &|n,m,\bar{m}\rangle_{matter} \otimes |n,m,\bar{m}\rangle_{in} \end{aligned} \quad (29)$$

This equation shows that matter and inside states of closed strings in evaporating black holes are entangled .This entanglement decreases for massive black holes and high energy strings.

IV. THE BLACK HOLE FINAL STATE FOR CLOSED STRINGS

Now we extend the Horowitz and Maldacena mechanism to closed strings. As mentioned above ,before black hole evaporation, there is an entanglement between the inside and outside of the horizon, but matter itself is not entangled.

$$|BH\rangle_{matter}^i \otimes_{in} \otimes_{out} = |BH\rangle_{matter}^i \otimes |BH\rangle_{in}^i \otimes_{out} \quad (30)$$

After evaporation, we can describe the state of the black hole as an entangled state of both matter and the inside of the black hole:

$${}_{matter \otimes in}^i \langle B\acute{H} | = {}_{matter \otimes in}^i \langle BH | (S \otimes I) \quad (31)$$

To describe the unknown effects of quantum gravity, an additional unitary transformation S is introduced[1, 9, 12]. The outside state of black hole can be described as:

$$\begin{aligned} & |BH\rangle_{out}^i = {}^i_{matter \otimes in} \langle BH | (S \otimes I) | BH \rangle_{matter \otimes in \otimes out}^i = \\ & \prod_n \frac{1}{\cosh^{2n} r_{E_s, \lambda}} \frac{1}{\cosh^{2n} r_{E_s, -\lambda}} \sum_{m, \bar{m}} \tanh^{2nm} r_{E_s, \lambda} \tanh^{2n\bar{m}} r_{E_s, -\lambda} \otimes \\ & {}_{matter} \langle n, m, \bar{m} | S | BH \rangle_{matter} \otimes | n, m, \bar{m} \rangle_{out} \end{aligned} \quad (32)$$

Until now, we have shown that the internal stationary state of the black hole for closed strings can be represented by a maximally entangled two-mode squeezed state of collapsing matter and infalling string radiation. The outgoing string radiation is obtained by the final state projection on the total wave function, which looks like a quantum teleportation process without the classical information transmitted. The black hole evaporation process as observed by the observer outside the event horizon is a unitary process but non-locality is required to transmit the string outside the event horizon. The uniqueness of the final state for closed strings seems to prevent any information from getting stuck there, so that all of the string information encoded in the initial collapsing black hole can appear in the outgoing string radiation. However Gottesman and PresKill believe that interactions between the incoming Hawking radiation and the collapsing matter can spoil the unitary nature of the black-hole evaporation [5], destroying some or all of the quantum information inside the hole [14].

V. HM MECHANISM AT CORRESPONDENCE POINT

After the first emission, the mass of the black hole is reduced to $M_{BH} - E_s$ due to energy conservation. Consequently after n emissions the mass of black hole is reduced to $M_{BH} - nE_s$. According to the string theory of quantum gravity, the minimum mass above which a black hole can be treated general relativistically is [2, 15]:

$$M_{min} \sim \frac{M_s}{g_s^2} \quad (33)$$

The properties of a black hole with mass M_{min} matches those of a string ball with the same mass. This is called the correspondence principle and the mass at which this happens is the correspondence point [2–4]. At this point that black hole makes a transition to a string, it can become a single string, multiple string, or radiation[2]. At this point, the entropy of

black hole and excited string should be the same[16]. At this stage we intend to work with entropy, to compare final states in black holes as well as in string balls at corresponding point. First we write down the density matrices ρ for the closed strings, using the direct product of $|BH\rangle_{out}$ and ${}_{out}\langle BH|$. This gives the density matrices for closed strings :

$$\rho = |BH\rangle_{out}\langle BH| = \prod_n \frac{1}{\cosh^{4n} r_{E_s, \lambda}} \frac{1}{\cosh^{4n} r_{E_s, -\lambda}} \otimes \sum_{m, \bar{m}} e^{-16\pi M_{BH} n m E_s} e^{-16\pi M_{BH} n \bar{m} E_s} |n, m, \bar{m}\rangle_{out} \langle n, m, \bar{m}| \quad (34)$$

We calculate the entropy S of the quantum states using the prescription of von Neumann [17], which makes direct use of the density matrix ρ . It is simply written as:

$$S = -Tr(\rho \ln \rho) \quad (35)$$

where the trace Tr is taken over the quantum states. We now apply these definitions to the entropy for the closed strings. After some mathematical manipulations for high energy strings, we obtain:

$$S_{BH} = - \sum_{n=1} \sinh^2 r_{E_s, \lambda} \ln \sinh^2 r_{E_s, \lambda} - \sum_{n=1} \sinh^2 r_{E_s, -\lambda} \ln \sinh^2 r_{E_s, -\lambda} \quad (36)$$

$$\sinh^2 r_{E_s, \lambda} = \frac{e^{-4\pi M_{BH} E_s - 2n\lambda}}{1 - e^{-4\pi M_{BH} E_s - 2in\lambda}} = \frac{e^{-\frac{E_s}{T_{BH}} - 2in\lambda}}{1 - e^{-\frac{E_s}{T_{BH}} - 2in\lambda}} \quad (37)$$

For $\lambda = 0$ the black hole entropy can be written as:

$$S_{BH} = -2 \sum_{n=1} \sinh^2 r_{E_s} \ln \sinh^2 r_{E_s} \quad (38)$$

where the Hawking temperature is given by[18]:

$$T_{BH} = \frac{1}{4\pi M_{BH}} \quad (39)$$

To consider the closed string states, a copy of the original Hilbert space may be constructed with a set of operators of creation/annihilation that have the same commutation properties as the original ones. The total Hilbert space is the tensor product of the two spaces H_{right} , H_{left} , where in this case $H_{left/right}$ denotes the physical quantum states space of the closed string. The left hand of string affects the right hand of string by entanglement[6, 10]. Thus,

in agreement with Refs. [6, 10] we may write the Bogoliubov transformation between the Minkowski and string creation and annihilation operators :

$$(\cosh r_{T_{sb}} \alpha_n^i - \sinh r_{T_{sb}} \bar{\alpha}_n^{i\dagger}) |\text{closed string}, n\rangle = 0 \quad (40)$$

$$(\cosh r_{T_{sb}} \bar{\alpha}_n^i - \sinh r_{T_{sb}} \alpha_n^{i\dagger}) |\text{closed string}, n\rangle = 0 \quad (41)$$

$$\tanh r_{T_{sb}} = e^{\frac{-E_s}{T_{sb}}} \quad (42)$$

The Hagedorn temperature of an excited string matches the Hawking temperature of a black hole at the correspondence point. The Hagedorn temperature is given by[2–4]:

$$T_{sb} = \frac{M_s}{\sqrt{8\pi}} \quad (43)$$

We can rewrite equations (40) and (41) as the following:

$$(\cosh r_{T_{sb}} \frac{\partial}{\partial \alpha_n^{i\dagger}} - \sinh r_{T_{sb}} \bar{\alpha}_n^{i\dagger}) |\text{closed string}, n\rangle = 0 \quad (44)$$

$$(\cosh r_{T_{sb}} \frac{\partial}{\partial \bar{\alpha}_n^{i\dagger}} - \sinh r_{T_{sb}} \alpha_n^{i\dagger}) |\text{closed string}, n\rangle = 0 \quad (45)$$

The solution of these equations is given by:

$$|\text{closed string}, n\rangle = \frac{1}{\cosh^2 r_{T_{sb}}} \sum_{m, \bar{m}=0}^{\infty} \tanh^{m+\bar{m}} r_{T_{sb}} |m\rangle \otimes |\bar{m}\rangle \quad (46)$$

Thus we can calculate the total ground state for closed strings as :

$$|\text{closed string}\rangle = \prod_n |\text{closed string}, n\rangle \quad (47)$$

The closed string entropy can be calculated as :

$$\begin{aligned} S_{sb} = & - \sum_{n=1} \alpha_n^{i\dagger} \alpha_n^i \ln \sinh^2 r_{T_{sb}} - \sum_{n=1} \alpha_n^i \alpha_n^{i\dagger} \ln \cosh^2 r_{T_{sb}} \\ & - \sum_{n=1} \bar{\alpha}_n^{i\dagger} \bar{\alpha}_n^i \ln \sinh^2 r_{T_{sb}} - \sum_{n=1} \bar{\alpha}_n^i \bar{\alpha}_n^{i\dagger} \ln \cosh^2 r_{T_{sb}} \end{aligned} \quad (48)$$

where

$$\sinh^2 r_{T_{sb}} = \frac{e^{\frac{E_s}{T_{sb}}}}{1 - e^{\frac{E_s}{T_{sb}}}} \quad (49)$$

By setting $T_{SB} = T_H$, for high energy strings, the entropy operator is obtained as :

$$\begin{aligned} S_{sb} = & - \sum_{n=1} \alpha_n^{i\dagger} \alpha_n^i \ln \sinh^2 r_{E_s, \lambda=0} - \sum_{n=1} \alpha_n^i \alpha_n^{i\dagger} \ln \cosh^2 r_{E_s, \lambda=0} \\ & - \sum_{n=1} \bar{\alpha}_n^{i\dagger} \bar{\alpha}_n^i \ln \sinh^2 r_{E_s, \lambda=0} - \sum_{n=1} \bar{\alpha}_n^i \bar{\alpha}_n^{i\dagger} \ln \cosh^2 r_{E_s, \lambda=0} \end{aligned} \quad (50)$$

To obtain the expectation value of closed excited string entropy, we calculate the expectation value of annihilation/creation operators of closed strings:

$$\begin{aligned} \langle closedstring, n | \alpha_n^{i\dagger} \alpha_n^i | closedstring, n \rangle = \\ \langle closedstring, n | \bar{\alpha}_n^{i\dagger} \bar{\alpha}_n^i | closedstring, n \rangle = \sinh^2 r_{E_s} \end{aligned} \quad (51)$$

Now we can obtain the expectation value of string entropy as following:

$$\langle S_{sb} \rangle = -2 \sum_{n=1} \sinh^2 r_{E_s} \ln(\sinh^2 r_{E_s}) = S_{BH} \quad (52)$$

This entropy is approximately equal to the black hole entropy with the same mass.

The correspondence principle offered a unique opportunity to test the Horowitz and Maldacena mechanism by calculating the black hole entropy at corresponding point and comparing it with the string ball entropy. We calculated the final outside state of black hole by applying Horowitz and Maldacena idea and obtained the entropy that carried out by these states. We found that this entropy matches the string ball entropy at corresponding point. This means that our result is consistent with correspondence principle and thus Horowitz and Maldacena mechanism works.

VI. INFORMATION LOSS FOR CLOSED STRING IN BLACK HOLES

The information loss in a black hole has been a serious challenge to modern physics, because it contradicts the basic principles of quantum mechanics. This argument predicts that a process of black hole formation and evaporation is not unitary. Gottesman and Preskill explained how interactions between the collapsing body and infalling Hawking radiation inside the event horizon could cause a loss of information. Gottesman and Preskill introduce a unitary transformation U to describe interactions between matter and the inside of the horizon [5]. In their fanciful language, the transformation U can be interpreted as an interaction between the informations past and future. They obtained the amount of this information loss for particles [5]. Using their method we calculate the information transformation from collapsing matter to outside of black hole as:

$$\begin{aligned} T_{closed\ string} =_{matter \otimes in} \langle B\dot{H} | BH \rangle_{in \otimes out} = \\ \prod_n \frac{1}{\cosh^{2n} r_{E_s, \lambda}} \frac{1}{\cosh^{2n} r_{E_s, -\lambda}} \sum_{m, \bar{m}} \tanh^{2nm} r_{E_s, \lambda} \tanh^{2nm} r_{E_s, -\lambda} \otimes _{matter} \langle m, \bar{m}, n |_{in} \langle m, \bar{m}, n | S \otimes U | m, \bar{m}, n \rangle_{in} | m, \bar{m}, n \rangle_{out} \end{aligned} \quad (53)$$

$$\begin{aligned}
f &= |T_{\text{closed string}}|^2 = \prod_n \left| \frac{1}{\cosh^{4n} r_{E_s, \lambda}} \frac{1}{\cosh^{4n} r_{E_s, -\lambda}} \right| \otimes \\
&\quad \sum_{m, \bar{m}, \dot{m}, \dot{\bar{m}}} e^{-8\pi n M_{BH} \omega_n (m + \dot{m})} e^{-8\pi n M_{BH} \omega_n (\bar{m} + \dot{\bar{m}})} \otimes \\
&\quad {}_{out} \langle \dot{m}, \dot{\bar{m}}, n | {}_{in} \langle \dot{m}, \dot{\bar{m}}, n | {}_{matter} \langle m, \bar{m}, n | {}_{in} \langle m, \bar{m}, n | S S^\dagger \otimes \\
&\quad U U^\dagger | m, \bar{m}, n \rangle {}_{in} | m, \bar{m}, n \rangle {}_{out} | \dot{m}, \dot{\bar{m}}, n \rangle {}_{in} | \dot{m}, \dot{\bar{m}}, n \rangle {}_{matter} = \\
&\quad \prod_n \left| \frac{1}{\cosh^{4n} r_{E_s, \lambda}} \frac{1}{\cosh^{4n} r_{E_s, -\lambda}} \right| \sum_{m, \bar{m}} e^{-16\pi n M_{BH} E_s m} e^{-16\pi n M_{BH} E_s \bar{m}} = \\
&\quad \prod_n \left(\frac{1 + e^{-16\pi M_{BH} E_s} - 2e^{-8\pi M_{BH} E_s} \cos 2n\lambda}{1 - e^{-16\pi M_{BH} E_s}} \right)^{2n} \quad (54)
\end{aligned}$$

$$S S^\dagger = I, U U^\dagger = I \quad (55)$$

If information transformation from the collapsing matter to the state of outgoing Hawking radiation is complete, then the value of f should be one [5, 14]. Thus, we observed that due to the deviation of the f from unity, some of information is lost. It seems that for all finite values of E_s , the value of f is not unity and so all information from all string emission processes experiences some degree of loss.

The purpose of these calculations is to examine the robustness of the escape of closed strings during black hole evaporation in final state projection models. In particular, we show that due to interactions between the matter and incoming Hawking radiation, all closed strings couldnt escape from the hole. Of strings that escape from the hole, low energy strings are lost, regardless of the number of strings in the hole to begin with. More precisely, the states of the strings that formed the hole are preserved under black hole evaporation with $f \simeq 1$ for high energy strings and with $f \simeq 0$ for low energy strings. These are the fidelity of escape of the string states of the collapsing matter: individual strings escape with a fidelity that approaches one as the energy of string in the hole becomes large. Since the fidelity is above the threshold for the use of quantum-error correcting codes, properly high energy strings can escape from the hole with fidelity arbitrarily close to 1.

VII. SUMMARY AND DISCUSSION

In this manuscript, we discuss the information loss in Schwarzschild black holes for closed strings. We calculate the amount of information transformation from collapsing matter to

outside of black hole by extending the Gottesman and Preskil method to string theory .We also test the HM mechanism at correspondence point. Using this idea we obtain the final state of black hole and calculate the entropy of these states. Then we compare this entropy with string ball entropy and observe that these entropies are the same at transition point and thus HM proposal works.

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